(6 pages) **Reg. No. : ....**

Code No. : 6303 **Sub. Code : PMAM 11/ ZMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics — Core

ALGEBRA – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer:

1. Let G be the group of integers under addition and let N be the set of all multiples of 3. Then $O(G/N)$ is

   (a) 1 (b) 2

   (c) 3 (d) infinity

2. If ϕ is a homomorphism of G in to $\overline{G}$, the kernel of ϕ is a subgroup of

(a) G (b) $\overline{G}$

(c) $G \times \overline{G}$ (d) $\overline{G} \times G$

3. Let G be a group of order 36. Suppose that G has a subgroup H of order 9. Then $i(H)$ is

(a) 4 (b) 45

(c) $9i + 36$ (d) 9

4. Let G be a group; for $g \in G$, the inner automorphism T_g is defined by

(a) $x\,T_g = xg$ (b) $x\,T_g = g^{-1}x\,g$

(c) $x\,T_g = gx$ (d) $x\,T_g = xgg^{-1}$

5. If $\theta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$ and $\Psi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ then $\theta\Psi$ is

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$

6. The value of $P(5)$ is

(a) 6 (b) 5

(c) 7 (d) 10

7. If $O(G) = 72$, the number of 3 – sylow subgroups of G is

(a) either 1 or 3 (b) either 1 or 4

(c) exactly 1 (d) either 1 or 4 or 8

8. The order of a 3 – sylow subgroup of a group of order 18 is

(a) 9 (b) 18

(c) 3 (d) 6

9. The number of non isomorphic abelian groups of order 3^4 is

(a) 4 (b) 3^4

(c) 5 (d) 3

10. If G is an abelian group and s is any integer then $G(s)$ is defined by

(a) $\{x \in G / x^s = e\}$

(b) $\{x \in G / o(x) = s\}$

(c) $\{x \in G / x^s = x\}$

(d) $\{x \in G / x^s \neq e\}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If H and K are two subgroups of G, define HK and give an example to show that HK need not be a subgroup of G.

Or

(b) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

12. (a) Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $0(a) > 0$, prove that $0(\phi(a)) = 0(a)$.

Or

(b) Define a solvable group and show that a subgroup of a solvable group is solvable.

13. (a) Prove that A_n is a normal subgroup of index 2 in S_n.

Or

(b) If $O(G) = P^2$ where P is a prime number, show that G is abelian.

14. (a) State and prove the second part of sylow's theorem.

Or

(b) Prove that any group of order $11^2 - 13^2$ must be abelian.

15. (a) Suppose that G is the internal direct product of $N_1, N_2 \ldots, N_n$. Then for $i \neq d$, Prove that $N_i \cap N_j = (e)$ and if $a \in N_i,$, $b \in N_j$, then $ab = ba$.

Or

(b) If G and G' are isomorphic abelian groups, then for every integer s, show that $G(s)$ and $G'(s)$ are isomorphic.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let H and K be subgroup of G, Prove that HK is a subgroup of G if and only if $HK = KH$.

Or

(b) Let ϕ be a hormomorphism of G onto $\overline{G}$ with kernel k. Prove that $G/K \approx \overline{G}$.

17. (a) State and prove Cayley's theorem.

Or

(b) Prove that $\vartheta(G) \approx G/Z$, where $\vartheta(G)$ is the group of inner automorphisms of G and Z is the center of G.

18. (a) If $O(G) = p^n$ where p is a prime number, prove that $Z(G) \neq (e)$.

Or

(b) Prove that the number of conjugate classes in S_n is $P_{(n)}$, the number of partitions of n.

19. (a) State and prove the third part of Sylow's theorem.

Or

(b) If G is a group of order 231. Prove that the 11-Sylow subgroup is in the center of G.

20. (a) Define the internal and external direct product of normal subgroups and show that they are isomorphic.

Or

(b) Show that every finite abelian group is the direct product of cyclic groups.

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